The semileptonic decays $B_c^* \to \eta_c \ell \bar{\nu}_\ell$ with QCD sum rules

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Abstract

In this article, we calculate the $B_c^* \to \eta_c$ form-factors with the three-point QCD sum rules, then study the semileptonic decays $B_c^* \to \eta_c \ell \bar{\nu}_\ell$, the predictions can be confronted with the experimental data in the future at the LHCb.

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1 Introduction

The bottom-charm quarkonium states are of special interesting, the ground states B_c and B_c^* which lie below the BD, BD^*, B^*D, B^*D^* thresholds cannot annihilate into gluons, and decay weakly through $\bar{b} \to \bar{c}W^+$, $c \to sW^+$, $c\bar{b} \to W^+$ at the quark level, furthermore, the B_c^* mesons also have the radiative transitions $B_c^* \to B_c\gamma$. The $B_c^\pm$ mesons have measurable lifetime, while the $B_c^{*\pm}$ mesons would have widths less than a hundred KeV [1]. The semileptonic decays $B_c^\pm \to J/\psi \ell^\pm \bar{\nu}_\ell$, $B_c^+ \to J/\psi e^+ \bar{\nu}_e$ were used to measure the B_c lifetime and the hadronic decays $B_c^\pm \to J/\psi \pi^\pm$ were used to measure the B_c mass in $p\bar{p}$ collisions at the energy $\sqrt{s} = 1.96\,\mathrm{TeV}$ by the CDF and D0 collaborations [2, 3, 4, 5]. Now the average values are $\tau_{B_c} = (0.45 \pm 0.04) \times 10^{-12}\,s$ and $m_{B_c} = (6.277 \pm 0.006)\,\mathrm{GeV}$ from the Particle Data Group [6]. The B_c^* mesons have not been observed yet, but they are expected to be observed and their properties be studies in details at the large hadron collider (LHC). The LHC will be the world's most copious source of the b hadrons, and a complete spectrum of the b hadrons will be available through gluon fusion. In proton-proton collisions at $\sqrt{s} = 14\,\mathrm{TeV}$, the $b\bar{b}$ cross section is expected to be $\sim 500\mu b$ producing 10^{12} $b\bar{b}$ pairs in a standard year of running at the LHCb operational luminosity of $2 \times 10^{32}\mathrm{cm}^{-2}\mathrm{sec}^{-1}$ [7].

The semileptonic decays $b \to c\ell\bar{\nu}_\ell$ are excellent subjects in exploring the CKM matrix element V_{cb}, we can use both the exclusive and inclusive $b \to c$ transitions to study the CKM matrix element V_{cb}. The semileptonic and nonleptonic B_c-decays have been studied extensively [8], in those studies, we often encounter the $B_c \to P$, V form-factors, which are highly nonperturbative quantities and should be calculated by some nonperturbative theoretical approaches. In this article, we calculate the $B_c^* \to \eta_c$ form-factors with the three-point QCD sum rules, then take those form-factors as the input parameters to study the semileptonic decays $B_c^* \to \eta_c \ell \bar{\nu}_\ell$. The QCD sum rules is a powerful nonperturbative theoretical tool in studying the ground state hadrons, and has given a lot of successful descriptions of the hadron properties [9, 10]. There have been several works on the semileptonic B_c-decays with the three-point QCD sum rules [11, 12, 13, 14], while there does not exist work on the semileptonic B_c^*-decays.

The article is arranged as follows: we study the $B_c^* \to \eta_c$ form-factors using the three-point QCD sum rules in Sect.2; in Sect.3, we present the numerical results and discussions; and Sect.4 is reserved for our conclusions.

2 The $B_c^* \to \eta_c$ form-factors with QCD sum rules

We study the $B_c^* \to \eta_c$ form-factors with the three-point correlation function $\Pi_{\mu\nu}(p_1, p_2)$,

$$\Pi_{\mu\nu}(p_1,p_2) = i^2 \int d^4x d^4y e^{ip_2\cdot x} e^{-ip_1\cdot y} \langle 0|T\{J_5(x) j_\mu(0) J_\nu(y)\}|0\rangle \,, \quad (1)$$

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where

$$\begin{aligned} J_5(x) &= \bar{c}(x) i\gamma_5 c(x)\,, \\ j_\mu(0) &= \bar{c}(0)\gamma_\mu(1-\gamma_5)b(0)\,, \\ J_\nu(y) &= \bar{b}(y)\gamma_\nu c(y)\,, \end{aligned} \tag{2}$$

the pseudoscalar current $J_5(x)$ and vector current $J_\nu(y)$ interpolate the pseudoscalar meson η_c and vector meson B_c^*, respectively, the $j_\mu(0)$ is the transition chiral current sandwiched between the B_c^* and η_c mesons.

We can insert a complete set of intermediate hadronic states with the same quantum numbers as the current operators $J_5(x)$ and $J_\nu(y)$ into the correlation function $\Pi_{\mu\nu}(p_1,p_2)$ to obtain the hadronic representation [9]. After isolating the ground state contributions from the heavy mesons B_c^* and η_c , we get the following result,

$$\begin{aligned} \Pi_{\mu\nu}(p_1,p_2) &= \frac{\langle 0|J_5(0)|\eta_c(p_2)\rangle\langle\eta_c(p_2)|j_\mu(0)|B_c^*(p_1)\rangle\langle B_c^*(p_1)|J_\nu(0)|0\rangle}{(m_{B_c^*}^2-p_1^2)(m_{\eta_c}^2-p_2^2)}+\cdots\,, \\ &= \frac{f_{\eta_c}m_{\eta_c}^2 f_{B_c^*}m_{B_c^*}}{2m_c(m_{B_c^*}^2-p_1^2)(m_{\eta_c}^2-p_2^2)}\left\{-ig_{\mu\nu}(m_{B_c^*}+m_{\eta_c})A_1(q^2)+ip_{1\mu}p_{2\nu}\frac{A_+(q^2)+A_-(q^2)}{m_{B_c^*}+m_{\eta_c}}\right. \\ &\quad \left. +ip_{2\mu}p_{2\nu}\frac{A_+(q^2)-A_-(q^2)}{m_{B_c^*}+m_{\eta_c}}-\epsilon_{\mu\nu\alpha\beta}p_1^\alpha p_2^\beta\frac{2V(q^2)}{m_{B_c^*}+m_{\eta_c}}+\cdots\right\}+\cdots\,, \end{aligned} \tag{3}$$

where we have used the following definitions for the $B_c^* \to \eta_c$ form-factors and decay constants of the B_c^* and η_c mesons,

$$\begin{aligned} \langle\eta_c(p_2)|j_\mu(0)|B_c^*(p_1)\rangle &= i\varepsilon_\mu(m_{B_c^*}+m_{\eta_c})A_1(q^2)+i(p_1+p_2)_\mu\varepsilon\cdot q\frac{A_+(q^2)}{m_{B_c^*}+m_{\eta_c}} \\ &\quad +iq_\mu\varepsilon\cdot q\frac{A_-(q^2)}{m_{B_c^*}+m_{\eta_c}}+\epsilon_{\mu\nu\alpha\beta}\varepsilon^\nu p_1^\alpha p_2^\beta\frac{2V(q^2)}{m_{B_c^*}+m_{\eta_c}}\,, \end{aligned} \tag{4}$$

$$\begin{aligned} \langle 0|J_\mu^\dagger(0)|B_c^*(p_1)\rangle &= f_{B_c^*}m_{B_c^*}\varepsilon_\mu\,, \\ \langle 0|J_5(0)|\eta_c(p_2)\rangle &= \frac{f_{\eta_c}m_{\eta_c}^2}{2m_c}\,, \end{aligned} \tag{5}$$

$q_\mu=(p_1-p_2)_\mu$, the ε_μ is the polarization vector of the B_c^* meson and satisfies the relation,

$$\sum_\lambda \varepsilon_\mu^*(\lambda,p)\varepsilon_\nu(\lambda,p) = -g_{\mu\nu}+\frac{p_\mu p_\nu}{p^2}\,. \tag{6}$$

In this article, we choose the tensor structures $g_{\mu\nu}$, $p_{1\mu}p_{2\nu}$, $p_{2\mu}p_{2\nu}$ and $\epsilon_{\mu\nu\alpha\beta}p_1^\alpha p_2^\beta$ to study the weak form-factors.

Here we will take a short digression to discuss the relations among the form-factors based on the heavy quark symmetry [15]. The $B_c^* \to \eta_c$ form-factors can be re-written as

$$\begin{aligned} \langle\eta_c(p_2)|j_\mu(0)|B_c^*(p_1)\rangle &= i\varepsilon_\mu(m_{B_c^*}+m_{\eta_c})A_1(q^2)+i(p_1+p_2)_\mu\varepsilon\cdot q\frac{A_2(q^2)}{m_{B_c^*}+m_{\eta_c}} \\ &\quad -2m_{B_c^*}iq_\mu\frac{\varepsilon\cdot q}{q^2}\left[A_3(q^2)-A_0(q^2)\right]+\epsilon_{\mu\nu\alpha\beta}\varepsilon^\nu p_1^\alpha p_2^\beta\frac{2V(q^2)}{m_{B_c^*}+m_{\eta_c}}\,, \end{aligned} \tag{7}$$

where

$$\begin{aligned} A_3(q^2) &= \frac{m_{B_c^*}+m_{\eta_c}}{2m_{B_c^*}}A_1(q^2)+\frac{m_{B_c^*}-m_{\eta_c}}{2m_{B_c^*}}A_2(q^2)\,, \\ A_+(q^2) &= A_2(q^2)\,,\ A_3(0)=A_0(0)\,, \\ A_-(q^2) &= -2m_{B_c^*}(m_{B_c^*}+m_{\eta_c})\frac{A_3(q^2)-A_0(q^2)}{q^2}\,. \end{aligned} \tag{8}$$

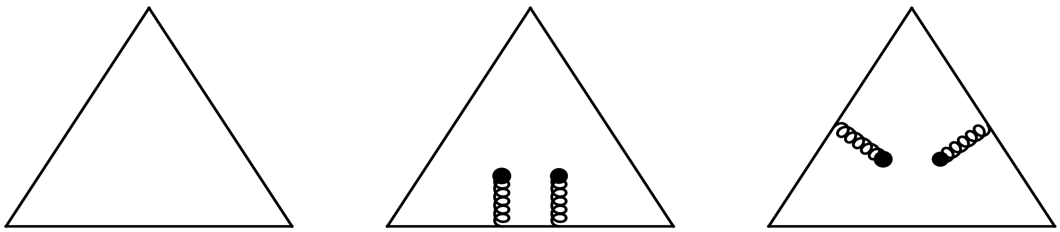

Figure 1: The typical diagrams we calculate in the operator product expansion, we take into account the tree-level perturbative term and gluon condensates.

In the heavy quark limit, the $B_c^* \to \eta_c$ form-factors can be parameterized by the universal Isgur-wise function $\xi(\omega)$,

$$\langle\eta_c(v')|j_\mu(0)|B_c^*(v)\rangle = i\left[\varepsilon_\mu(v\cdot v'+1)-v_\mu\varepsilon\cdot v'\right]\xi(\omega)+\epsilon_{\mu\nu\alpha\beta}\varepsilon^\nu v^\alpha v'^\beta\xi(\omega)\,, \tag{9}$$

where the v_μ and v'_μ are four-velocities, and $\omega = v\cdot v'$. Then we obtain the following relations,

$$V(q^2) = A_2(q^2)=A_0(q^2)=A_1(q^2)\left[1-\frac{q^2}{(m_{B_c^*}+m_{\eta_c})^2}\right]^{-1}=\frac{m_{B_c^*}+m_{\eta_c}}{2\sqrt{m_{B_c^*}m_{\eta_c}}}\xi(\omega)\,. \tag{10}$$

The vector state $|B_c^*(v)\rangle$ relates with the pseudoscalar state $|B_c(v)\rangle$ through $|B_c^*(v)\rangle = 2S_b^3|B_c(v)\rangle$, where the S_b^3 is the heavy quark spin operator. We can also express the $B_c \to \eta_c$ form-factors in terms of the Isgur-wise function $\xi(\omega)$,

$$\langle\eta_c(v')|j_\mu(0)|B_c(v)\rangle = \xi(\omega)(v+v')_\mu\,. \tag{11}$$

On the other hand, the $B_c \to \eta_c$ form-factors are usually parameterized by the two form-factors $F_1(q^2)$ and $F_0(q^2)$,

$$\langle\eta_c(p_2)|j_\mu(0)|B_c(p_1)\rangle = F_1(q^2)\left[(p_1+p_2)_\mu-\frac{m_{B_c}^2-m_{\eta_c}^2}{q^2}q_\mu\right]+F_0(q^2)\frac{m_{B_c}^2-m_{\eta_c}^2}{q^2}q_\mu\,. \tag{12}$$

The form-factors $F_1(q^2)$ and $F_0(q^2)$ relate with the Isgur-wise function $\xi(\omega)$ through,

$$F_1(q^2) = F_0(q^2)\left[1-\frac{q^2}{(m_{B_c}+m_{\eta_c})^2}\right]^{-1}=\frac{m_{B_c}+m_{\eta_c}}{2\sqrt{m_{B_c}m_{\eta_c}}}\xi(\omega)\,. \tag{13}$$

Finally we obtain the following relations among the $B_c^* \to \eta_c$ and $B_c \to \eta_c$ form-factors in the heavy quark limit,

$$V(q^2) = A_2(q^2)=A_0(q^2)=F_1(q^2)\,,\ A_1(q^2)=F_0(q^2)\,. \tag{14}$$

In the following, we briefly outline the operator product expansion for the correlation function $\Pi_{\mu\nu}(p_1,p_2)$ in perturbative QCD. We contract the quark fields in the correlation function $\Pi_{\mu\nu}(p_1,p_2)$ with Wick theorem firstly, replace the b and c quark propagators with the corresponding full propagators, then calculate all the Feynman diagrams with the Cutkosky's rule. We take into account the tree-level contributions and the gluon condensates in the operator product expansion, the typical Feynman diagrams are shown in Fig.1.

For example, the lowest order tree-level diagram can be calculated as

$$\begin{aligned}\Pi_{\mu\nu}(p_1,p_2) &= \frac{3}{(2\pi)^4}\int d^4k\frac{Tr\left\{\gamma_5\left[(\not k+\not p_2)+m_c\right]\gamma_\mu(1-\gamma_5)\left[(\not k+\not p_1)+m_b\right]\gamma_\nu\left[\not k+m_c\right]\right\}}{\left[(k+p_2)^2-m_c^2\right]\left[(k+p_1)^2-m_b^2\right]\left[k^2-m_c^2\right]}\,,\\ &= \int ds_1ds_2\frac{\rho(s_1,s_2,q^2)}{(s_1-p_1^2)(s_2-p_2^2)}\,,\end{aligned} \tag{15}$$

where

$$\rho(s_1,s_2,q^2) = -\frac{3i}{(2\pi)^3}\int d^4k\delta\left[(k+p_2)^2-m_c^2\right]\delta\left[(k+p_1)^2-m_b^2\right]\delta\left[k^2-m_c^2\right]$$
$$Tr\left\{\gamma_5\left[(\not k+\not p_2)+m_c\right]\gamma_\mu(1-\gamma_5)\left[(\not k+\not p_1)+m_b\right]\gamma_\nu\left[\not k+m_c\right]\right\}\,, \qquad (16)$$

we carry out the integral over the variable k to obtain the spectral density at the quark level. The Feynman diagrams for the gluon condensates like the typical ones shown in Fig.1 are calculated analogously.

The calculations are straightforward and tedious, in the following, we present the basic formulae used in this article,

$$\int d^4k\delta^3 = \frac{\pi}{2\sqrt{\lambda(s_1,s_2,q^2)}}\,, \qquad (17)$$
$$\int d^4k k_\mu\delta^3 = \frac{\pi}{2\sqrt{\lambda(s_1,s_2,q^2)}}\left[a_1p_{1\mu}+b_1p_{2\mu}\right]\,, \qquad (18)$$
$$\int d^4k k_\mu k_\nu\delta^3 = \frac{\pi}{2\sqrt{\lambda(s_1,s_2,q^2)}}\left[a_2p_{1\mu}p_{1\nu}+b_2p_{2\mu}p_{2\nu}+c_2(p_{1\mu}p_{2\nu}+p_{1\nu}p_{2\mu})+d_2g_{\mu\nu}\right]\,, \qquad (19)$$

where

$$\begin{aligned}
\delta^3 &= \delta[k^2-m^2]\delta[(k+p_1)^2-m_1^2]\delta[(k+p_2)^2-m_2^2]\,,\\
a_1 &= -\frac{\widetilde{s}_2(s_1+s_2-q^2)-2s_2\widetilde{s}_1}{\lambda(s_1,s_2,q^2)}\,,\\
b_1 &= -\frac{\widetilde{s}_1(s_1+s_2-q^2)-2s_1\widetilde{s}_2}{\lambda(s_1,s_2,q^2)}\,,\\
a_2 &= \frac{\widetilde{s}_2^2+2s_2m^2}{\lambda(s_1,s_2,q^2)}+6s_2\frac{s_1\widetilde{s}_2^2+s_2\widetilde{s}_1^2-\widetilde{s}_1\widetilde{s}_2(s_1+s_2-q^2)}{\lambda(s_1,s_2,q^2)^2}\,,\\
b_2 &= \frac{\widetilde{s}_1^2+2s_1m^2}{\lambda(s_1,s_2,q^2)}+6s_1\frac{s_1\widetilde{s}_2^2+s_2\widetilde{s}_1^2-\widetilde{s}_1\widetilde{s}_2(s_1+s_2-q^2)}{\lambda(s_1,s_2,q^2)^2}\,,\\
c_2 &= \frac{1}{s_1+s_2-q^2}\left\{\frac{2\widetilde{s}_1\widetilde{s}_2(s_1+s_2-q^2)-3(s_1\widetilde{s}_2^2+s_2\widetilde{s}_1^2)}{\lambda(s_1,s_2,q^2)}-m^2\left[1+\frac{4s_1s_2}{\lambda(s_1,s_2,q^2)}\right]\right.\\
&\left.-12s_1s_2\frac{s_1\widetilde{s}_2^2+s_2\widetilde{s}_1^2-\widetilde{s}_1\widetilde{s}_2(s_1+s_2-q^2)}{\lambda(s_1,s_2,q^2)^2}\right\}\,,\\
d_2 &= \frac{m^2}{2}+\frac{s_1\widetilde{s}_2^2+s_2\widetilde{s}_1^2-\widetilde{s}_1\widetilde{s}_2(s_1+s_2-q^2)}{2\lambda(s_1,s_2,q^2)}\,,
\end{aligned} \qquad (20)$$

$\widetilde{s}_i=s_i+m^2-m_i^2$, $i=1,2$, and $\lambda(a,b,c)=a^2+b^2+c^2-2ab-2bc-2ca$. The formulae in Eqs.(17-18) are consistent with that obtained in Refs.[16, 17], while the formula in Eq.(19) is slightly different from that of the Ref.[17].

Once the analytical expressions of the correlation function at the quark level are obtained, then we can take quark-hadron duality below the threshold s_1^0 and s_2^0 in the channels B_c^* and η_c respectively, take double Borel transform with respect to the variables $P_1^2=-p_1^2$ and $P_2^2=-p_2^2$

respectively, finally obtain four QCD sum rules for the weak form-factors,

$$
\begin{aligned}
A_1(q^2) &= \frac{2m_c}{f_{\eta_c}m_{\eta_c}^2 f_{B_c^*}m_{B_c^*}(m_{B_c^*}+m_{\eta_c})}\int ds_1 ds_2 \left\{\frac{3\mathcal{C}\left[m_c(s_1+s_2-q^2)+s_2(m_b-m_c)\right]}{8\pi^2\sqrt{\lambda(s_1,s_2,q^2)}}\right. \\
&\left.+\frac{3m_c(s_1-q^2)+2m_b s_2}{12\pi\lambda(s_1,s_2,q^2)^{\frac{3}{2}}}\langle\frac{\alpha_s GG}{\pi}\rangle\right\}\exp\left\{\frac{m_{B_c^*}^2-s_1}{M_1^2}+\frac{m_{\eta_c}^2-s_2}{M_2^2}\right\}, \\
\widetilde{A}_+(q^2) &= \frac{2m_c(m_{B_c^*}+m_{\eta_c})}{f_{\eta_c}m_{\eta_c}^2 f_{B_c^*}m_{B_c^*}}\int ds_1 ds_2\left\{\frac{3\mathcal{C}}{4\pi^2\sqrt{\lambda(s_1,s_2,q^2)}}\right. \\
&\left[m_c+(m_b-m_c)\frac{s_2^2-s_2(s_1+q^2+2m_c^2-2m_b^2)}{\lambda(s_1,s_2,q^2)}\right] \\
&+\frac{3s_1m_c}{2\pi\lambda(s_1,s_2,q^2)^{\frac{3}{2}}(s_1+s_2-q^2)}\left[1+\frac{4s_1s_2}{\lambda(s_1,s_2,q^2)}\right]\langle\frac{\alpha_s GG}{\pi}\rangle \\
&-\frac{m_c}{2\pi\lambda(s_1,s_2,q^2)^{\frac{3}{2}}(s_1+s_2-q^2)}\left[s_1+s_2+2q^2+\frac{12s_1s_2q^2}{\lambda(s_1,s_2,q^2)}\right]\langle\frac{\alpha_s GG}{\pi}\rangle \\
&+\frac{m_b}{3\pi\lambda(s_1,s_2,q^2)^{\frac{3}{2}}}\left[1+\frac{6s_1s_2}{\lambda(s_1,s_2,q^2)}\right]\langle\frac{\alpha_s GG}{\pi}\rangle-\frac{m_b}{3\pi\lambda(s_1,s_2,q^2)^{\frac{3}{2}}(s_1+s_2-q^2)} \\
&\left.\left[s_1-2s_2-q^2+\frac{6s_1s_2(s_1+s_2-q^2)}{\lambda(s_1,s_2,q^2)}\right]\langle\frac{\alpha_s GG}{\pi}\rangle\right\}\exp\left\{\frac{m_{B_c^*}^2-s_1}{M_1^2}+\frac{m_{\eta_c}^2-s_2}{M_2^2}\right\}, \\
\widetilde{A}_-(q^2) &= \frac{2m_c(m_{B_c^*}+m_{\eta_c})}{f_{\eta_c}m_{\eta_c}^2 f_{B_c^*}m_{B_c^*}}\int ds_1 ds_2\left\{\frac{3\mathcal{C}m_c\left[2s_1s_2-(s_1+m_c^2-m_b^2)(s_1+s_2-q^2)\right]}{2\pi^2\lambda(s_1,s_2,q^2)^{\frac{3}{2}}}\right. \\
&-\frac{3s_1^2m_c}{\pi\lambda(s_1,s_2,q^2)^{\frac{5}{2}}}\langle\frac{\alpha_s GG}{\pi}\rangle+\frac{m_c}{2\pi\lambda(s_1,s_2,q^2)^{\frac{3}{2}}}\left[1+\frac{6s_1q^2}{\lambda(s_1,s_2,q^2)}\right]\langle\frac{\alpha_s GG}{\pi}\rangle \\
&\left.-\frac{m_b}{3\pi\lambda(s_1,s_2,q^2)^{\frac{3}{2}}}\left[1+\frac{6s_1s_2}{\lambda(s_1,s_2,q^2)}\right]\langle\frac{\alpha_s GG}{\pi}\rangle\right\}\exp\left\{\frac{m_{B_c^*}^2-s_1}{M_1^2}+\frac{m_{\eta_c}^2-s_2}{M_2^2}\right\}, \\
V(q^2) &= \frac{m_c(m_{B_c^*}+m_{\eta_c})}{f_{\eta_c}m_{\eta_c}^2 f_{B_c^*}m_{B_c^*}}\int ds_1 ds_2\left\{\frac{3\mathcal{C}}{4\pi^2\sqrt{\lambda(s_1,s_2,q^2)}}\right. \\
&\left.\left[m_c+(m_b-m_c)\frac{s_2^2-s_2(s_1+q^2+2m_c^2-2m_b^2)}{\lambda(s_1,s_2,q^2)}\right]\right\}\exp\left\{\frac{m_{B_c^*}^2-s_1}{M_1^2}+\frac{m_{\eta_c}^2-s_2}{M_2^2}\right\},
\end{aligned}
\tag{21}
$$

where

$$
\begin{aligned}
\widetilde{A}_+(q^2) &= A_+(q^2)+A_-(q^2), \\
\widetilde{A}_-(q^2) &= A_+(q^2)-A_-(q^2), \\
\int ds_1 ds_2 &= \int_{(m_b+m_c)^2}^{s_1^0} ds_1\int_{4m_c^2}^{s_2^0} ds_2 \mid_{|2s_1s_2-(s_1+s_2-q^2)(s_1+m_c^2-m_b^2)|\leq\sqrt{\lambda(s_1,s_2,q^2)\lambda(s_1,m_c^2,m_b^2)}}, \\
\mathcal{C} &= \sqrt{\frac{4\pi\alpha_s^{\mathcal{C}}}{3v}\left[\frac{1}{1-\exp\left(-\frac{4\pi\alpha_s^{\mathcal{C}}}{3v}\right)}\right]}, \\
v &= \sqrt{1-\frac{4m_bm_c}{s_1-(m_b-m_c)^2}}.
\end{aligned}
\tag{22}
$$

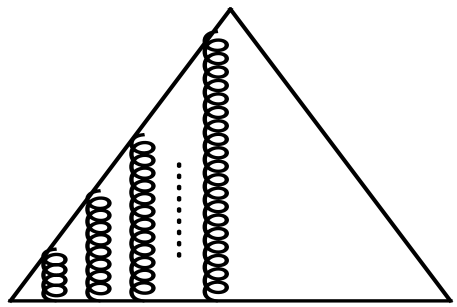

Figure 2: The ladder Feynman diagrams of the Coulomb-like interactions.

For the heavy quarkonium state B_c^*, the relative velocity of quark movement is small, we should account for the Coulomb-like $\frac{\alpha_s^{\mathcal{C}}}{v}$ corrections. After taking into account all the Coulomb-like contributions shown in Fig.2, we obtain the coefficient $\mathcal{C}$ to dress the quark-meson vertex, where the $\alpha_s^{\mathcal{C}} = 0.45$ for the $\bar{b}c$ (or $b\bar{c}$) systems [13].

3 Numerical results and discussions

The input parameters are taken as $m_{\eta_c} = 2.981\,\text{GeV}$ [6], $m_b = 4.7\,\text{GeV}$, $m_c = 1.3\,\text{GeV}$, $\langle\frac{\alpha_s GG}{\pi}\rangle = (0.33\,\text{GeV})^4$ [18], $s_1^0 = (45 \pm 1)\,\text{GeV}^2$, $f_{B_c^*} = 0.58\,\text{GeV}$, $m_{B_c^*} = 6.34\,\text{GeV}$ [19], $s_2^0 = (15 \pm 1)\,\text{GeV}^2$ [9], and $f_{\eta_c} = 0.35\,\text{GeV}$ [20]. In Fig.3, we plot the weak form-factors at $q^2 = 0$ with variations of the Borel parameters M_1^2 and M_2^2 respectively. From the figure, we can see that the form-factors decrease monotonously with the increase of the Borel parameters at the region $M_1^2 \leq 2.75\,\text{GeV}^2$ and $M_2^2 \leq 0.75\,\text{GeV}^2$, and no stable QCD sum rules can be obtained. In this article, we take the Borel parameters as $M_1^2 = (3.5-5.5)\,\text{GeV}^2$ and $M_2^2 = (0.9-1.7)\,\text{GeV}^2$, the values are rather stable with variations of the Borel parameters. The contributions from high resonances and continuum states are greatly suppressed, $\exp(-\frac{s_1^0}{M_1^2}) \leq e^{-8}$ and $\exp(-\frac{s_2^0}{M_2^2}) \leq e^{-8.2}$. If we choose much larger Borel parameters, the numerical values of the weak form-factors changes slightly, see Fig.3, the predictions still survive. The numerical values of the weak form-factors at zero momentum transfer are

$$\begin{aligned}
A_1(0) &= 0.25 \pm 0.01\,, \\
A_+(0) &= 0.25 \pm 0.03\,, \\
A_-(0) &= 0.52 \pm 0.02\,, \\
V(0) &= 0.39 \pm 0.02\,.
\end{aligned} \tag{23}$$

From Eq.(21), we can also obtain the numerical values of the weak form-factors at the squared momentum q^2, then fit them to an exponential form,

$$f(q^2) = f(0)\exp\left(c_1 q^2 + c_2 q^4\right)\,, \tag{24}$$

where the $f(q^2)$ denote the weak form-factors $A_1(0)$, $A_+(0)$, $A_-(0)$ and $V(0)$, the c_1 are c_2 are fitted parameters. The numerical values of the fitted parameters c_1 and c_2 are presented in Table 1.

The calculations based on the three-point QCD sum rules indicate that the $B_c \to \eta_c$ form-factor $F_1(0)$ is 0.20 ± 0.02 from Ref.[11], 0.55 ± 0.10 from Ref.[12], 0.66 from Ref.[13], the differences are rather large, as very different input parameters are taken in those studies. If the heavy quark spin symmetry works well, the present work indicates that $A_1(0) = A_2(0) = F_1(0) = 0.25$, which is compatible with the value 0.20 from Ref.[11]. While in the present work $A_1(0) = A_2(0) < V(0)$

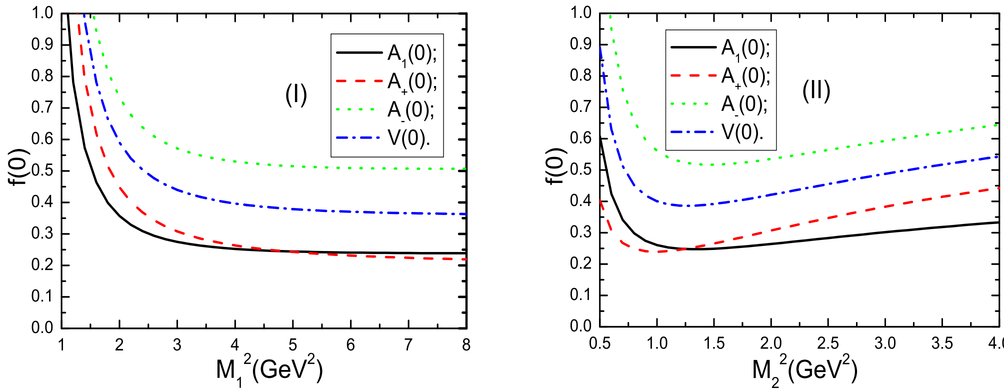

Figure 3: The weak from-factors with variations of the Borel parameters M_1^2 and M_2^2, where $M_2^2 = 1.3\,\mathrm{GeV}^2$ in (I) and $M_1^2 = 4.5\,\mathrm{GeV}^2$ in (II).

with the difference $\frac{V(0)-A_1(0)}{V(0)} = 36\%$, the heavy quark spin symmetry works not well enough, as the c quark mass is not large enough.

The semileptonic decays $B_c^* \to \eta_c \ell \nu_\ell$ can be described by the effective Hamiltonian $\mathcal{H}_{\rm eff}$,

$$\mathcal{H}_{\rm eff} = \frac{G_F}{\sqrt{2}} V_{cb} \bar{c}\gamma_\alpha(1-\gamma_5) b\, \bar{\nu}_\ell \gamma^\alpha (1-\gamma_5)\ell\,, \tag{25}$$

where the V_{cb} is the CKM matrix element and the G_F is the Fermi constant. We take into account the effective Hamiltonian $\mathcal{H}_{\rm eff}$ and the weak form-factors $A_1(q^2)$, $A_+(q^2)$, $A_-(q^2)$ and $V(q^2)$ to obtain the squared amplitude $|T|^2$,

$$|T|^2 = 4G_F^2 V_{cb}^2 (l^\alpha v^\beta + l^\beta v^\alpha - l\cdot v g^{\alpha\beta}) \langle \eta_c(p)|j_\alpha(0)|B_c^*(P)\rangle \left[\langle \eta_c(p)|j_\beta(0)|B_c^*(P)\rangle\right]^\dagger\,, \tag{26}$$

where the P, p, l and v are the four-momenta of the B_c^*, η_c, ℓ and $\bar{\nu}_\ell$, respectively. Finally we obtain the differential decay widths,

$$d\Gamma = \sum \frac{|T|^2}{6m_{B_c^*}} \frac{dq^2}{2\pi} d\Phi(P\to q,p)\, d\Phi(q\to l,v)\,, \tag{27}$$

where the $d\Phi(P\to q,p)$ and $d\Phi(q\to l,v)$ are the two-body phase factors defined analogously, for example,

$$d\Phi(P\to q,p) = (2\pi)^4\delta^4(P-q-p)\frac{d^3\vec{p}}{(2\pi)^3 2p_0}\frac{d^3\vec{q}}{(2\pi)^3 2q_0}\,. \tag{28}$$

We take the revelent parameters as $G_F = 1.166364\times 10^{-5}\,\mathrm{GeV}^{-2}$, $V_{cb} = 40.6\times 10^{-3}$, $m_e = 0.510998928\,\mathrm{MeV}$, $m_\mu = 105.6583715\,\mathrm{MeV}$, $m_\tau = 1776.82\,\mathrm{MeV}$ from the Particle Data Group [6], then obtain the differential decay widths and decay widths,

$$\begin{aligned}
\Gamma(B_c^* \to \eta_c e\bar{\nu}_e) &= 5.41\times 10^{-15}\,\mathrm{GeV}\,,\\
\Gamma(B_c^* \to \eta_c \mu\bar{\nu}_\mu) &= 5.39\times 10^{-15}\,\mathrm{GeV}\,,\\
\Gamma(B_c^* \to \eta_c \tau\bar{\nu}_\tau) &= 2.13\times 10^{-15}\,\mathrm{GeV}\,,
\end{aligned} \tag{29}$$

the numerical values of the differential decay widths $d\Gamma/dq^2$ are shown in Fig.4. The present predictions can be confronted with the experimental data at the LHCb in the future.

$A_1(0)$ $[c_1/c_2]$	$A_+(0)$ $[c_1/c_2]$	$A_-(0)$ $[c_1/c_2]$	$V(0)$ $[c_1/c_2]$
0.25 [0.109/0.00062]	0.25 [0.125/0.00085]	0.52 [0.133/0.00096]	0.39 [0.130/0.00093]

Table 1: The parameters for the weak form-factors.

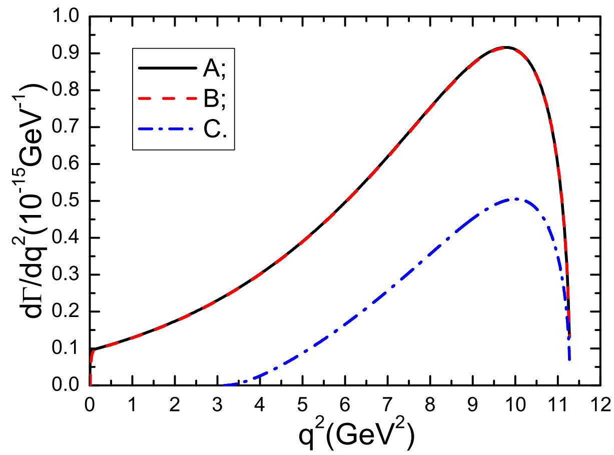

Figure 4: The differential decay widths with variations of the squared momentum q^2, the A, B and C denote $d\Gamma(B_c^* \to \eta_c e\bar{\nu}_e)/dq^2$, $d\Gamma(B_c^* \to \eta_c \mu\bar{\nu}_\mu)/dq^2$ and $d\Gamma(B_c^* \to \eta_c \tau\bar{\nu}_\tau)/dq^2$, respectively.

4 Conclusion

In this article, we study the $B_c^* \to \eta_c$ form-factors with the three-point QCD sum rules, then take those weak form-factors as the basic input parameters to calculate the semileptonic decay widths and differential decay widths. The vector B_c^* mesons and the semileptonic decays $B_c^* \to \eta_c \ell\bar{\nu}_\ell$ have not been observed yet, the present predictions can be confronted with the experimental data in the future at the LHCb.

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